[redacted]/Ph. D Entrance Exam Syllabus ~~2016~~ 2021

Part ~~A~~ B

1. **Set Theory :**

   Cartesian product, Functions, Binary Relations, Equivalence Relations, Partitions, Partial Order Relation, Finite sets, countable and uncountable sets.

2. **Logic :**

   Logical connectives, Tautologies, Contradictions, Logical equivalence, Truth tables, Algebra of propositions, Quantifiers, Boolean Algebra, De Morgans Laws,

3. **Real Analysis & Complex Analysis :**

   Real number system as a complete ordered field, Archimedean property, Supremum, Infimum, Limit, Continuity, Differentiability, Maclaurin & Taylor series, Definition of a sequence, Theorems on limits of sequences, Bounded and monotonic sequences, Cauchy's convergence criterion, Series of non-negative terms, Comparison test, Ratio test, Leibnitz's theorem, Absolute convergence, The fundamental theorem of integral calculus, Mean value theorem of integral calculus, Vector differentiation, Gradient, Divergence and Curl, Applications of Gauss, Green's & Stoke's theorems, Algebra of complex numbers, The complex plane, Polynomials, Power series, Continuity and Differentiability of a function of a complex variable, Analytical functions, Cauchy Riemann equations, Harmonic functions, Mobius transformations,

4. **Metric spaces :**

   Definition and examples of metric spaces, Neighbourhoods, Limit points, Interior points, Open and closed sets, Closure and interior, Boundary points, Subspace of a metric space, Cauchy sequences, Completeness, Cantor's intersection theorem.

5. **Algebra :**

   Pigeon-hole principle, Inclusion-exclusion principle, Fundamental theorem of arithmetic, Divisibility in the ring of integers, congruences, Groups, Sub groups, Permutation groups, Cyclic groups, Lagrange's theorem and its consequences, Normal subgroups, Quotient groups, Group homomorphism, Kernel of a homomorphism, Fundamental theorem of homomorphism of groups, Group isomorphism, Cayley's theorem, Rings, Ideals, Maximal ideals, Prime ideals, Domains & fields, Ring homomorphism, Ring isomorphism & related theorems, Quotient rings.

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6. **Linear Algebra & Matrix Theory :**

Vector spaces, Sub spaces, linearly dependent & linearly independent vectors, Basis, Dimension, linear transformation, Matrix representation of a linear transformation, Rank & Nullity theorem. Finite dimensional vector spaces, Existence theorem for basis, Quotient space and its dimension. Rank of a matrix, Eigen values & Eigen vectors, Cayley Hamilton theorem, Inverse of a matrix, Solutions of system of linear equations.

7. **Differential equations :**

Ordinary differential equations and its solutions, Partial differential equations & its solutions, Linear Differential equations and equations reducible to the linear form, Exact differential equations, First order and higher degree equations, Linear differential equations with constant coefficients. Homogenous linear ordinary differential equations, linear differential equations of second order.

Part B

1. **Real Analysis :**

Bolzano-Weierstrass theorem, Heine Borel theorem, Continuity, Uniform continuity, Differentiability, mean value theorem, sequences & series of functions, Point wise convergence, limsuperior, limit inferior, Uniform convergence, Riemann sums and Riemann integral, Improper integrals, Monotonic functions, types of discontinuity, functions of bounded variations, Lebesgue measure, Lebesgue integral.

2. **Topolgy :**

Compactness, Connectedness of metric spaces, Basics of topology, Subspace and Product topology, Separation axioms, Connectedness, Compactness of topological spaces.

3. **Functional Analysis :**

Normed linear spaces, Inner product spaces, Orthonormal basis, Spaces of continuous functions, Quotient space, Conjugate space, Banach spaces, Riesz Fischer theorem, Hahn Banach extension theorem, Open mapping theorem, Uniform boundedness principle & its applications, Hilbert spaces, Riesz representation theorem, Projections, Invariant subspaces.

theorem, Rings, Ideals, Maximal ideals, Prime ideals, Domains & fields, Ring homomorphism, Ring isomorphism & related theorems, Quotient rings.

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4. **Linear Algebra:**

Change of basis, Canonical forms, Diagonal forms, Triangular forms, Jordan forms, Quadratic forms, reduction and classification of quadratic forms, Orthogonal transformations, Unitary transformations, Positive semi definite matrices, Semi definite matrices.

5. **Complex Analysis :**

Transcendental functions such as exponential, Trigonometric and Hyperbolic functions, Contour integrals, Cauchy's theorem, Cauchy's integral formula, Liouville's theorem, Maximum modulus principle, Schwarz lemma, Open mapping theorem, Taylor series, Laurent series, calculus of residues, conformal mappings.

6. **Algebra :**

Group automorphisms, Inner automorphism, Group of automorphisms, Conjugacy relation and centraliser, Normaliser, Counting principle and the class equation of a finite group, Cauchy's theorem, Sylow theorems,

Chinese Remainder theorem, Euler's phi function, primitive roots, Unique Factorization Domains, Principal Ideal Domains, Euclidean domains, Polynomial rings, irreducibility criteria, Basic concepts related to extension of fields.

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