Devi Ahilya Vishwavidyalaya, Indore

Ph.D./M.Phil. Entrance Test

Syllabus for Subject: Mathematics

Part A

This Part shall consist of 50 objective type compulsory questions of 1 mark each based on research methodology. It shall be of generic nature, intended to assess the research aptitude of the candidate. It will primarily be designed to test reasoning ability, data interpretation, and quantitative aptitude of the candidate.

Part B

1. **Real Analysis :**

Real number system as a complete ordered field, Archimedean property, Supremum, Infimum, Limit, Continuity, Differentiability, Maclaurin & Taylor series, Definition of a sequence, Theorems on limits of sequences, Bounded and monotonic sequences, Cauchy's convergence criterion, Series of non-negative terms, Comparison test, Ratio test, Leibnitz's theorem, Absolute convergence.

Bolzano-Weierstrass theorem, Heine Borel theorem, Continuity, Uniform continuity, Differentiability, mean value theorem, sequences & series of functions, Point wise convergence, limit superior, limit inferior, Uniform convergence, Riemann sums and Riemann integral, Improper integrals, Monotonic functions, types of discontinuity, functions of bounded variations, Lebesgue measure, Lebesgue integral.

2. **Topolgy :**

Definition and examples of metric spaces, Neighbourhoods, Limit points, Interior points, Open sets, closed sets, Closure and interior, Boundary points, Subspace of a metric space, Cauchy sequences, Completeness, Cantor's intersection theorem.

Compactness, Connectedness of metric spaces, Basics of topology, Subspace and Product topology, Separation axioms, Connectedness, Compactness of topological spaces.

3. **Functional Analysis :**

Normed linear spaces, Inner product spaces, Orthonormal basis, Spaces of continuous functions, Quotient space, Conjugate space, Banach spaces, Riesz Fischer theorem, Hahn Banach extension theorem, Open mapping theorem, Uniform boundedness principle & its applications, Hilbert spaces, Riesz representation theorem, Projections, Invariant subspaces.

4. **Linear Algebra:**

Vector spaces, Sub spaces, linearly dependent & linearly independent vectors, Basis, Dimension, linear transformation, Matrix representation of a linear transformation, Rank & Nullity theorem. Finite dimensional vector spaces, Existence theorem for basis, Quotient space and its dimension. Rank of a matrix, Eigen values & Eigen vectors.

Change of basis, Canonical forms, Diagonal forms, Triangular forms, Jordan forms, Quadratic forms, reduction and classification of quadratic forms, Orthogonal transformations, Unitary transformations, Positive semi definite matrices, Semi definite matrices.

5. **Complex Analysis :**

Algebra of complex numbers, The complex plane, Polynomials, Power series, Continuity and Differentiability of a function of a complex variable, Analytical functions, Cauchy Riemann equations, Harmonic functions, Mobius transformations.

Transcendental functions such as exponential, Trigonometric and Hyperbolic functions, Contour integrals, Cauchy’s theorem, Cauchy’s integral formula, Liouville’s theorem, Maximum modulus principle, Schwarz lemma, Open mapping theorem, Taylor series, Laurent series, calculus of residues, conformal mappings.

6. **Algebra :**

Divisibility in the set of integers, congruences, Groups, Sub groups, Permutation groups, Cyclic groups, Lagrange's theorem and its consequences, Normal subgroups, Quotient groups, Group homomorphism, Kernel of a homomorphism, Fundamental theorem of homomorphism of groups, Group isomorphism, Cayley’s theorem, Rings, Ideals, Maximal ideals, Prime ideals, Domains & fields, Ring homomorphism, Ring isomorphism & related theorems, Quotient rings.

Polynomial rings, irreducibility criteria, Basic concepts related to extension of fields. Structure of modules over Principal Ideal Domains.